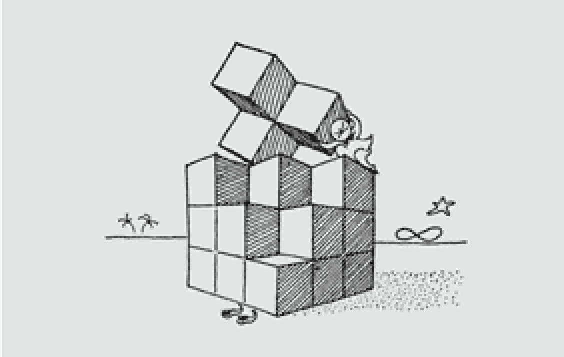

*"... no time, no leisure ... not a moment to sit down and think - or if ever by some unlucky chance such a crevice of time should yawn in the solid substance of their distractions, there is always soma, delicious soma..."*

Aldous Huxley, Brave New World

From time to time efforts have been made to devise a puzzle in three dimensions. None, in my opinion, has been as successful as the Soma cube, invented by Piet Hein, the Danish writer whose mathematical games, Hex and Tac Tix, are discussed in the first Scientific American Book of Mathematical Puzzles. In Denmark, Piet Hein is best known for his books of epigrammatic poems written under the pseudonym Kumbel.)

Piet Hein conceived of the Soma cube during a lecture an quantum physics by Werner Heisenberg. While the noted German physicist was speaking of a space sliced into cubes, Piet Hein's supple imagination caught a fleeting glimpse of the following curious geometrical theorem. If you take all the irregular shapes.that can be formed by combining no more than four cubes, all the same size and joined at their faces, these shapes can be put together to form a larger cube.
Let us make this clearer. The simplest irregular shape - "irregular" in the Sense that it has a concavity or corner nook in it somewhere - is produced by joining three cubes as shown at 1 in Figure I. It is the only such shape possible with three cubes. (Of course no irregular shape is possible with one or two cubes.) Turning to four cubes, we find that there are six different ways to form irregular shapes by joining the cubes face to face. These are pieces 2 to 7 in the illustration. To identify the seven pieces Piet Hein labels them with numerals. No two shapes are alike, although 5 and 6 are mirror images of each other. Piet Hein points out that two cubes can be joined only along a single coordinate, three cubes can add a second coordinate perpendicular to the first, and four cubes' are necessary to supply the third coordinate perpendicular to the other two. Since we cannot enter the fourth dimension to join cubes along a fourth coordinate supplied by five-cube shapes, it is reasonable to limit our set of Soma pieces:to seven. It is an unexpected fast that these elementary combinations of identical cubes can be joined to form a cube again.

As Heisenberg talked on, Piet Hein swiftly convinced himself by doodling on a sheet of paper that thee seven pieces, containing 27 small cubes, would form a 3 x 3 x 3 cube. After the lecture he glued 27 cubes into the shapes of the seven compönents and quickly confirmed his insight. A set of the pieces was marketed under the trade name Soma, and the puzzle has since become a popular one in the Scandinavian countries.

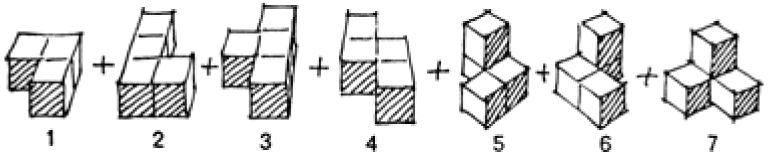


Fig. I

As a first lesson in the art of Soma, see if you can combine any two pieces to form the stepped structure in Figure II.

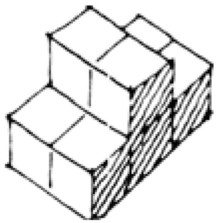

Fig. II

Having mastered this trivial problem, try assembling all seven pieces into a cube. It is one of the easiest of all Soma constructions. More than 230 essentially different solutions (not counting rotations and reflections) have been tabulated by Richard K. Guy of the University of Malaya, in Singapore, but the exact number of such solutions has not yet been determined. A good strategy to adopt an this as well as other Soma figures is to set the more irregular shapes (5, 6 and 7) in place first, because the other pieces adjust more readily to remaining gaps in a structure. Piece 1 in particular is best saved until last.

After solving the cube, try your hand at the more difficult seven-piece structures in Figure III. Instead of using a time-consuming trial and error technique, it is much more satisfying to analyze the constructions and cut down your building time by geometrical insights. For example, it is obvious that pieces 5, 6 and 7 cannot form the steps to the well. Group competition can be introduced by giving each player a Soma set and seeing who can build a given figure in the shortest length of time. To avoid misinterpretations of these structures it should be said that the far sides of the pyramid and steamer are exactly like the near sides ; both the hole in the well and the interior of the bathtub have a volume of three cubes; there are no holes or projecting pieces an the hidden sides of the skyscraper; and the column that forms the back of the dog's head consists of four cubes, the bottom one of which is hidden from view.

After working with the pieces for several days, many people find that the shapes become so familiar that they can solve Soma Problems in their heads. Tests made by European psychologists have shown that ability to solve Soma Problems is roughly correlated with general intelligence, but with peculiar discrepancies at both ends of the I.Q. curve. Some geniuses are very poor at Soma and some morons seem specially gifted with the kind of spatial imagination that Soma exercises. Everyone who takes such a test wants to keep playing with the pieces after the test is over.

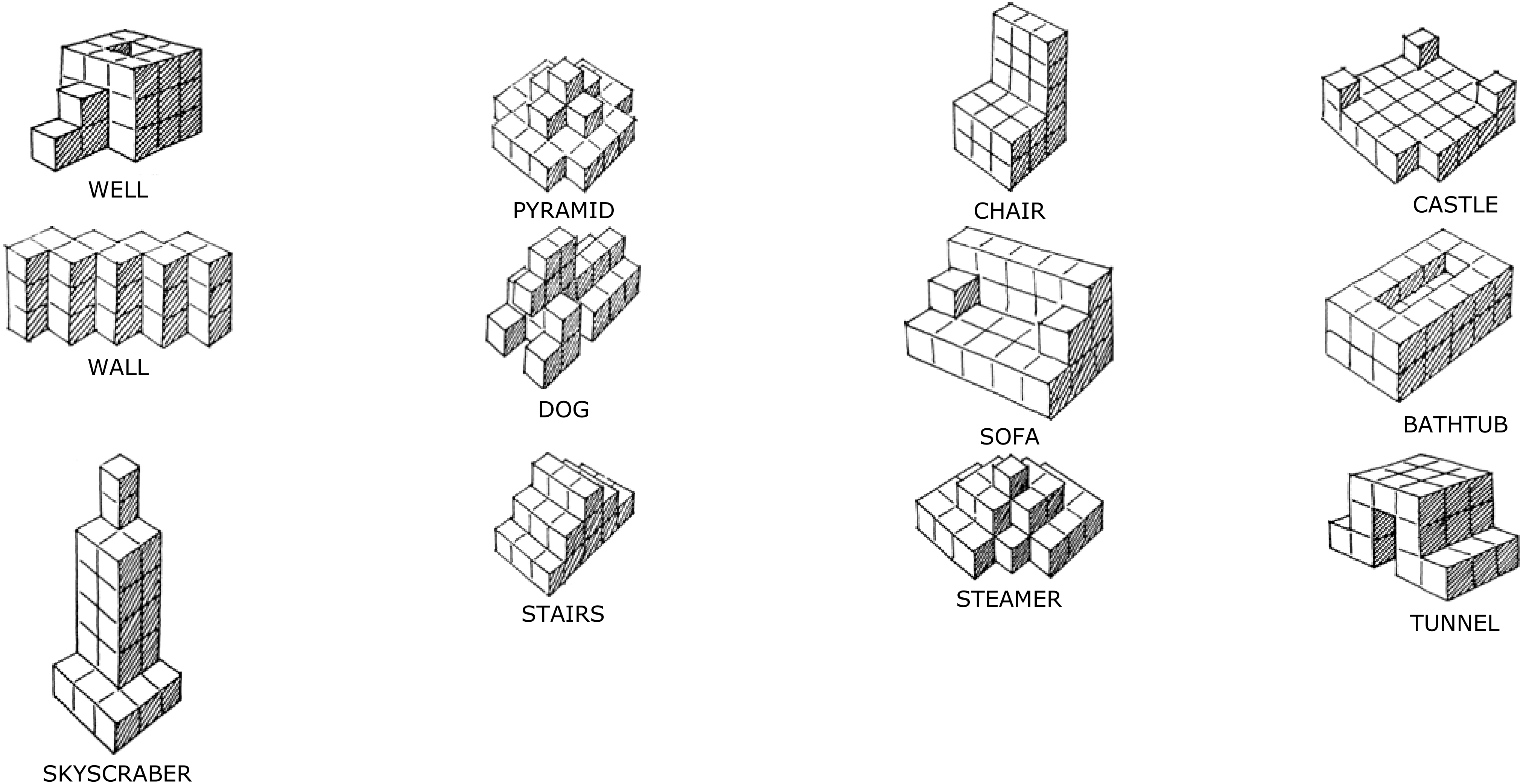


Fig. III

Like the two-dimensional polyominoes, Soma constructions lend themselves to fascinating theorems and impossibility proofs of combinatorial geometry. Consider the structure in the top illustration of Figure IV. No one had succeeded in building it, but it was not until recently that a formal impossibility proof was devised. Here is the clever proof, discovered by Solomon W. Golomb, mathematician at the Jet Propulsion Laboratory of the California Institute of Technology.

We begin by looking down an the structure as shown in the bottom illustration and coloring the columns in checker-board fashion. Each column is two cubes deep except for the center column, which consists of three cubes. This gives us a total of eight white cubes and 19 black, quite an astounding disparity.

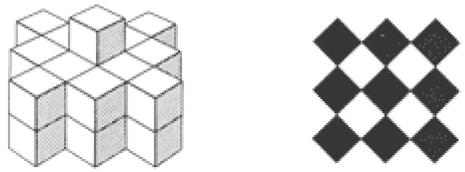

Fig. IV

The next step is to examine each of the seven components, testing it in all possible orientations to ascertain the maxi-mus number of black cubes it can possess if placed within the checkerboard structure. The chart in Figure V displays this maximum number for each piece. As you see, the total is 18 black to nine white, just one short of the 19-8 split demanded. If we shift the top black block to the top of one of the columns of white blocks, then the black-white ratio changes to the required 19-8, and the structure becomes possible to build.

I must confess that one of the structures in Figure III is impossible to make. It should take the average reader many days, however, to discover which one it is. Methods for building the other figures will not be given in the answer section (it is only a matter of time until you succeed with any one of them), but I shall identify the figure that cannot be made.

| SOMA PIECE | MAXIMUM BLACK CUBES | MINIMUM WHITE CUBES |
|---|---|---|
| 1. | 2 | 1 |
| 2. | 3 | 1 |
| 3. | 3 | 1 |
| 4. | 2 | 2 |
| 5. | 3 | 1 |
| 6. | 3 | 1 |
| 7. | 2 | 2 |
| | 18 | 9 |

Fig. V

The number of pleasing structures that can be built with the seven Soma pieces seems to be as unlimited as the number of plane figures that can be made with the seven tangram shapes. It is interesting to note that if piece 1 is put aside, the remaining six pieces will form a shape exactly like 1 but twice as high.

ADDENDUM

When i wrote the column about Soma, I supposed that few readers would go to the trouble of actually making-a set. I was wrong. Thousands of readers sent sketches of new Soma figures and many complained that their leisure time had been obliterated since they were bitten by the Soma bug. Teachers made Soma sets for their classes. Psychologists added Soma to their psychological tests. Somaddicts made sets for friends in hospitals and gave them as Christmas gifts. A dozen firms inquired about manufacturing rights. Gem Color Company, 200 Fifth Avenue, New York, N.Y., marketed a wooden set - the only set authorized by Piet Hein - and it is still selling in toy and novelty stores.

From the hundreds of new Soma figures received from readers, I have selected the twelve that appear in Figure VI.
Some of these figure's were discovered by more than one reader. All are possible to construct.
The charm of Soma derives in Part, I think, from the fact that only seven pieces are used; one is not overwhelmed by complexity. All sorts of variant Sets, with a larger number of pieces, suggest themselves, and I have received many letters describing them.

ANSWERS

THE ONLY structure in Figure III that is impossible to construct with the seven Soma pieces is the skyscraper.

*

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